A STUDY ON THE BOUNDARY LAYER CHARACTERISTICS OF NANOFLUIDS PAST STRETCHING SURFACES

SYNOPSIS

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1 Introduction

Fluid mechanics is most important and oldest branch of mechanics, which deals with the behavior of fluids at rest and in motion. The study of the behavior of fluids at rest is termed as fluid static where as study of fluids at motion is termed as fluid dynamics. It is very necessary to have a working knowledge of fluid mechanics in order to modeling the red spot on Jupiter, to find the vorticity generated in a tornado, in order to design the ships, jets, rockets, etc. Fluid mechanics laid the foundation stones for various fields of sciences and engineering like meteorology, oceanology, aerodynamics, bio-fluids, propulsion and combustion, particles physics and acoustics. Fluid dynamics is considered as a progressive field of research due to its wide range of applications in mathematics, physics, engineering, etc. In this research work, some fluid flow phenomena are taken into consideration. The fundamental principles of fluid dynamics are conservation of mass, linear momentum, angular momentum, energy, and second law of thermodynamics. Navier-Stokes equations are used to define the motion of a viscous fluid and these equations are highly nonlinear in nature. Due to the highly nonlinear nature of these equations, it is very difficult to obtain the exact solution of these equations except for limited cases. Exact solutions are available only for limited cases only after eliminating viscous terms. But at the beginning of nineteenth century, an achievement of Ludwig Prandtl in this direction set forth a way to find the solutions of N-S equations without eliminating nonlinear terms[1].

The concept of boundary layer theory was developed by Ludwig Prandtl[2] in 1904. In his experiment he exhibited that flow past an object can be split into two sections namely, (i) a thin layer in the neighborhood of the object where the impact of viscosity in prominent called boundary layer and (ii) a region outside this thin layer where viscosity effect can be ignored. This exceptionally useful concept of Prandlt paved a way to reduce the difficulty to solve Navier-Stokes equations to a larger extent and explained the importance of viscosity in viscous flows. The calculations of drags forces of flat plate, drag of ship, the drag of an airplane, etc. are highly important applications of Prandtl boundary layer theory .Boundary layer theory is very helpful to calculate pressure drag of the object, which is created due to the change in pressure distributions because of flow separation. Boundary layer theory is also very helpful to determine the shape of the object in order to avoid flow separation.

Fundamental equations of fluid flows: The basic equations for the analysis of behavior of fluids are based on some fundamental principles of fluid mechanics and these equations are: equation of state, equation of continuity, equation of motion(Navier-Stokes equations) and equation of energy.

For the flow of viscous compressible fluid, these equations are respectively stated in tensor form a below:

Equation of state

$$p = \rho RT,$$

Equation of continuity

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_j) = 0,$$

Equation of motion(Navier-Stokes equations)

$$\rho\left(\frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x_j}\right) = \rho f_b - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left\{\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}\right)\right\},$$

Equation of energy

$$\rho\frac{\mathbf{D}}{\mathbf{Dt}}(c_pT) = \frac{\mathbf{D}}{\mathbf{Dt}}p + \frac{\partial Q}{\partial t} + \frac{\partial}{\partial x_i}\left(\kappa\frac{\partial T}{\partial x_i}\right) + \phi,$$

where

$$\frac{\mathbf{D}}{\mathbf{Dt}} = \left(\frac{\partial}{\partial t} + v_j\frac{\partial}{\partial x_j}\right) \text{ is the material deriative,}$$

and

$$\phi = \mu\left\{\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}\right)\right\}\cdot\frac{\partial v_i}{\partial x_j}$$

is the viscous dissipative heat and $i, j = 1, 2, 3$. Other symbols have usual meanings.

For viscous incompressible fluid, the above equations take the following form : Equation of state becomes redundant (i.e ρ = constant),

Equation of continuity

$$\frac{\partial v_j}{\partial x_j} = 0,$$

Equation of motion (Navier-Stokes equations)

$$\rho\left(\frac{\partial v_i}{\partial t}+v_j\frac{\partial v_i}{\partial x_j}\right)=\rho f_b-\frac{\partial p}{\partial x_i}+\mu\frac{\partial^2 v_i}{\partial x_j\partial x_j},$$

Equation of energy

$$\rho c_p\frac{\mathbf{D}}{\mathbf{Dt}}(T)=\frac{\partial Q}{\partial t}+\kappa\frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right)+\phi,$$

where,

$$\phi=\mu\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)\frac{\partial v_i}{\partial x_j}\text{ and }i,j=1,2,3$$

The transfer of heat from one point to other point is the prime concern in many industrial and technological applications. Improving heat transfer efficiency is most important in the telecommunication system. Electronic cooling system exhaust more than fifty percent of the total electrical energy. The high heat dissipation rate of order of $2000W/cm^2$ is demanded in an electronic system like optical devices, X-rays and laser applications. Initially, the performances of the system and devices were improved by using water, engine oils, ethylene glycol, formamide, etc. But to maintain more efficiency of these devices, we required technologies with advanced heat transfer rates. Hence, small solid particles were added in carrier liquids because they can produce sufficient heat as compared to liquids. The major problems with the use of these particles were rapid setting, abrasion and clogging in fluids. These problems are highly repellent for many practical cooling applications. Nowadays some latest innovations explained the usage of nanoparticles in heat transfer applications in place of micrometers sized particles. Nanoparticles are fall in range of size 1-100 nanometers. Nanoparticles possess higher surface area and surface/volume ratio of nano particles is 1000 times greater than that of micro-sized particles. Generally, nanoparticles are made from metals and their oxides. Commonly known nanoparticles are single-wall nanoparticles, multiwall nanoparticles, iron, silver, copper, aluminum, and their oxides. It was Choi[3] and Eastman who pioneered this marvelous idea of suspending the nanoparticles in a regular fluid. Thermal conductivity of metallic particles and metal oxides are higher than regular fluids, hence greater is the heat transfer.

Boundary layer flow over stretching surfaces play a prominent role due to its vast applications in different manufacturing processes in industries such as extraction

of polymer sheets, textile industries, coating industries, glass fiber production, paper production, drawing of plastic sheets, hot rolling, movements of biological fluids, the performance of lubricants and paints, and so on. The steady boundary layer flow of incompressible viscous flow over a linearly stretching plate was firstly investigated by Crane [4]. This effort of Crane was further continued by Gupta and Gupta [5], Chen and Char[6], and Pavlov [7] under various physical conditions. Vajravelu [8] and Cortell [9] explain the boundary layer flows over a non-linear streching sheet. It was Hiemenz [10] who studied the behavior of steady flow in the neighborhood of a stagnation point. Chiam [11] combined the work of Hiemenz and Crane i.e. the stagnation point flow a stretching sheet when the stretching rate of the plate is equal to the stain rate and no boundary layer structure was found across the plate. Exceptional attempts have been made to obtain information regarding the stretching flow problem in different physical situations including magnetic fluids, porous surfaces, slip effects, suction, injection, etc. Magyari and Keller [12] added a new dimension in these investigations by studying the fluid flow characteristics over an exponentially stretching sheet. Mukhopadhyay[13] studied the slip flow effects on MHD flow past an exponential stretching sheet. Elbashbeshy[14] investigated the heat transfer rate over a permeable exponential stretching surface. Several attempts have been made to describe the flow characteristics of fluids towards exponentially stretching surfaces.

According to the Buongiorno[15] model, the equations for nanofluid past a stretching sheet are:
The continuity equation for the nanofluid is:

$$\nabla.v = 0$$

where v is the nanofuid velocity and is identical to the continuity equation for a pure incompressible fluid. The continuity equation for the nanoparticles in the absence of chemical reactions is:

$$\frac{\partial \phi}{\partial t} + v.\nabla\phi = -\frac{1}{\rho_p}\nabla.\mathbf{j}_p$$

where t is time, $\mathbf{j}_p$ is the diffusion mass flux for the nanoparticles (Kg/m^2s), and represents the nanoparticle flux relative to the nanofluid velocity v. If the external forces are negligible, $\mathbf{j}_p$ can be written as the sum of only two diffusion terms, i.e., Brownian

Diffusion and thermophoresis:

$$\mathbf{j}_p = \mathbf{j}_{p,B} + \mathbf{j}_{p,T} = -\rho_p D_B \nabla\phi - \rho_p D_T \frac{\nabla T}{T}$$

Now putting the values of diffusion coefficients D_B , D_T, and $\mathbf{j}_p$ in continuity equation, we get

$$\frac{\partial\phi}{\partial t} + v.\nabla\phi = \nabla.\left[D_B\nabla\phi + D_T\frac{\nabla T}{T}\right]$$

This equation states that the nanoparticles can move homogeneously with the fluid(second term of the left-hand side), but they also possess a slip velocity relatively to the fluid(right hand side), which is due to Brownian diffusion and thermophoresis.

The momentum equation for the nanofluid with negligible external forces is:

$$\rho\left[\frac{\partial v}{\partial t} + v.\nabla v\right] = -\nabla P - \nabla.\tau$$

where P is pressure and this equation is identical to the momentum equation for a pure fluid and τ is stress tensor.

The energy equation for the nanofluid can be written as:

$$\rho c\left[\frac{\partial T}{\partial t} + v.\nabla T\right] = -\nabla.\mathbf{q} + h_p\nabla.\mathbf{j}_p$$

where c is the nanofluid specific heat, T is the nanofluid temperature, h_p is the specific enthalpy of the nanoparticle material(J/Kg), and $\mathbf{q}$ is the energy flux relative to the nanofluid velocity v.

The flow influenced by a stretching boundary has wide applications in the suspension processes in metal and plastic industries. Sakiadis drifted out a instigate analysis and studied the boundary layer flow on a continuously stretching surface with a constant speed. The Power-law velocity function was enforced at the stretching surface and the thermal boundary conditions incorporated a power-law surface temperature or a power-law surface heat flux. The mass transfer effect was also examined on the stretching surface. Lee[16] was the first who studied on Boundary layer over a thin body with variable thickness passing a needle with variable diameters. Then, comprehensive studies were conducted on the Boundary layer flows over a thin needle considering heat transfer effects. However,

considering a variable sheet thickness, there is no investigation of the non-flatness effects on the stretching sheet problems. The variable thickness stretching sheet may occur more frequently in the engineering applications than a flat sheet. For a sheet with a variable stretching speed at different locations, depending on the extensibility or the stretchability of the sheet material, the sheet thickness can change with the motion of the sheet. For example, for a linearly stretching sheet of an incompressible material, if the stretching speed is linearly proportional to the distance from the slot, the sheet thickness decreases linearly with the distance. For other materials with different extensibility, the sheet thickness may change according to other profiles. A stretching sheet with a variable thickness can be more close to the situation in practical applications.

2 Literature Review

Sakiadis [17] studied the boundary layer flow on continuous solid surfaces. He observed that the flow of this type represents a new class of boundary layer problems, with solutions substantially different from those for boundary layer flow on surfaces of finite length.

Turcotte [18] developed a theory for a finite amplitude, steady cellular convection. The theory requires that the Prandtl number be large compared with unity and that the Rayleigh number be large compared with the critical Rayleigh number and considered only two-dimensional laminar cells. The core of each cell is an isothermal, highly viscous rotating flow. Thin thermal boundary layers are formed on the horizontal boundaries. When the thermal boundary layers from adjacent cells meet they separate from the horizontal boundary and form a thermal plume on the vertical boundary between cells. The body force in the plumes drives the viscous core flow. It is found that the Nusselt number for the total heat transfer between the horizontal boundaries is proportional to the Rayleigh number to the one quarter power.

Shamroth and McDonald [19] investigated this problem to assess the capability of a finite difference boundary layer procedure to predict the mean profile development across a transition from laminar to turbulent flow in the low hypersonic Mach number regime. He showed the ability of this procedure to predict Stanton number, velocity profiles and density profiles through the transition region, and, in addition, to predict the effect of

wall cooling and Mach number upon transition Reynolds number. He also examined the contribution of the pressure dilation term to the energy balance and suggests that transition can be initiated from the direct absorption of acoustic energy even if only a small amount (1 percent) of the incident acoustic energy is absorbed.

Ryzhov and Savenkov [20] considered the propagation of a wave packet generated by a point source in a boundary layer on a flat plate. The fluid is assumed to be incompressible, and the distance from the leading edge of the plate was chosen to be so large, so that the Reynolds number can be assumed to tend infinity and he constructed the field of perturbed motion by using the framework of the linearized theory of the boundary layer with self induced pressure.

Pakdemirli and Suhubi [21] have derived the boundary layer equations for second order fluids using a suitable coordinate system in which the equations are independent of the body shape immersed into the flow and concluded that the general results for different flow types may be deduced by using the same equations.

Yimin and Qiang [22] formulated a procedure for preparing a nanofluid which is a suspension of nanophase powders and a base liquid. He introduced the theoretical study of thermal conductivity of nanofluids. The nanofluids shows great potential in enhancing the heat transfer process due to the suspended ultra-fine particles that remarkably increase the thermal conductivity of the nanofluid. The volume fraction, shape, dimensions and properties of the nanoparticles affects the thermal conductivity of the nanofluids. The hot-wire method has been used to measure the thermal conductivity of the nanofluids. He concluded that the thermal conductivity of nanofluids remarkably increases with the volume fraction of ultra-fine particles.

Putra et al. [23] studied the heat transfer characteristics of nano-fluids under natural convection inside a horizontal cylinder heated and cooled from the two ends respectively. The fluid appears to behave more homogenously compared to common slurries. Unlike conduction or forced convection, a systematic and definite deterioration in natural convection heat transfer has been found to occur. The deterioration is independent on particle density, concentration as well as the aspect ratio of the cylinder.

Jang and Choi [24] have analyzed that Brownian motion of nanoparticles at the molecular and nanoscale levels is a key nanoscale mechanism governing their thermal

behavior. It has been observed that there is a connection between measured macroscale phenomena and molecular/nanoscale fluctuations.

Hayat et al. [25] have analyzed the magneto hydrodynamic boundary layer flow of a second grade fluid over a shrinking sheet. It has been found that for the series solution, the governing nonlinear problem is solved using the homotopy analysis method and the convergence of the obtained solution is analyzed explicitly.

Wang [26] studied the stagnation flow towards a shrinking sheet. He considered the both two-dimensional and axisymmetric stagnation flows. It has been observed that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case. The non-alignment of the stagnation flow and the shrinking sheet complicates the flow structure. Convective heat transfer decreases with the shrinking rate due to an increase in boundary layer thickness.

Khan and Pop [27] investigated numerically the problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid. He used a model for nanofluid that incorporates the effects of Brownian motion and thermophoresis. It has been observed that the reduced Nusselt number is a decreasing function of each dimensionless number, while the reduced Sherwood number is an increasing function of higher Pr and a decreasing function of lower Pr number for each Le, Nb and Nt numbers.

Gan and Qiao [28] quantitatively determined the radiative properties of various nano-fluid fuels and their effects on droplet evaporation. He concluded that radiation absorption could be significantly enhanced by adding a small amount of nanoparticles such as Al to the base fluid like ethanol. The strong absorption of radiation energy by nanoparticles increases the nanofluid droplet temperatures and enhances droplet evaporation rate.

Amin and Das [29] have analyzed the Unsteady boundary layer development over a flat plate under the influence of wake vortices induced from an elliptic cylinder was investigated for different Reynolds number. The interaction between the boundary layer developed over a flat plate and the cylinder wake has been studied to visualize the effect of the presence of a cylinder in the laminar boundary layer over a flat surface, where the cylinder controls the laminar boundary layer flow. Two types of boundary layer-wake interaction were obtained from the computations, one is the strong wake-boundary layer

interaction and another is the weak wake-boundary layer. The intensity of interaction greatly depends on plate stream-wise location and the Reynolds number. The higher the Reynolds number, the more rapidly the wake loses its coherence, and the more unstable the boundary layer is to perturbation. Despite the difference in the Reynolds number, the mechanism of interaction is the same for all the cases.

Yacob et al. [30] investigated numerically the characteristics of fluid flow and heat transfer over a stretching vertical sheet immersed in a nanofluid. He studied three different types of nanoparticles, namely, copper (Cu), alumina (Al_2O_3), and titania (TiO_2), are considered, by using water as the base fluid. He found that nanofluid with titania nanoparticles has better enhancement on the heat transfer rate as compared to copper and aluminia nanoparticles. For a particular nanoparticle, increasing the nanoparticle fraction is to reduce the skin friction coefficient and the heat transfer rate at the surface.

Pal et al. [31] numerically studied the effects of thermal radiation and viscous dissipation on hydromagnetic mixed convection flow over a non-linear stretching and shrinking sheets in nanofluids. It is found that the velocity and thermal fields are influenced appreciably by the magnetic field, effective Prandtl number, local Grashof number, Eckert number, power-law stretching parameter and nanoparticles volume fraction. He presented the results for local skin-friction and local Nusselt number and also dual solutions for velocity and temperature fields for various values of magnetic field and suction/injection parameters.

Hacka and Zaki [32] examined the influence of span wise wall oscillation on transition in boundary layers exposed to free-stream turbulence. It is observed that for moderate amplitudes of the wall motion, transition to turbulence can be considerably delayed. On the other hand, high amplitude forcing leads to breakdown to turbulence upstream of the unforced reference flow configuration. Flow visualizations and statistical results show that the stabilization of the flow at the optimal forcing parameters is the result of a substantial weakening of the low-frequency boundary layer streaks. On the other hand, the early breakdown at high forcing amplitudes is associated with the amplification of new high-frequency instabilities in the pre-transitional flow.

Abbas et al. [33] deals with the laminar fluid of a viscous flow and heat transfer analysis due to a porous stretching/shrinking cylinder. It is found that the dual exist for the shrinking

cylinder, while a unique solution exists for a stretching cylinder.

Selvakumar and Dhinakaran [34] investigated the steady, laminar and 2-D nano-fluid flow around a circular cylinder to show the effects of variable viscosity on flow characteristics. He used a comparative analysis of the three cases shows that the flow characteristics of nanofluids become unpredictable due to the uncertainties in effective viscosity. In the first case, nanofluids show an accelerated flow with either flow separation or longer wake bubbles. Whereas, in the other two cases, a decelerated flow with delayed flow separation is noted. He concluded that the flow characteristics of nanofluid vary both qualitatively and quantitatively due to the variations in effective viscosity.

Wu [35] studied the effect of mass-transfer induced velocity slip on heat transfer of laminar boundary layer gas flows above a linearly stretching/shrinking sheet at macroscale. It is found that mass-transfer induced velocity slip is able to appreciably change the gas flow velocity, which subsequently leads to non-negligible changes in temperature distribution and wall heat flux through thermal convection process.

Ramya et al. [36] studied numerically the boundary layer viscous flow of nanofluids and heat transfer over a non-linearly stretching sheet in the presence of a magnetic field. He analyzed the influence of the governing parameters on the dimensionless velocity, temperature, nano particle concentration as well as the skin friction coefficient, Nusselt number and local Sherwood number. He found that as the velocity slip parameter increases, the velocity profile is decreased and the skin friction and heat transfer decreased while the mass transfer is increased. Increasing the thermal slip parameter causes decreases in the heat and mass transfer rates.

Ghosh and Mukhopadhyay [37] presented the steady boundary layer flow of nanofluid past an exponentially porous shrinking sheet in the presence of heat and mass fluxes. He considered the combined effects of Brownian motion and thermophoresis on heat transfer and nanoparticle volume fraction. His investigation reveals that the variable heat flux and mass flux have major significant effects on temperature field and the nanoparticle volume fraction. The wall mass transfer through the porous sheet causes an increase of fluid velocity for the first branch of solution. Temperature as well as nanoparticles volume fraction decreases for both branches of solutions. For the Brownian motion, the temperature increases but the nanoparticle volume fraction decreases. Heat transfer rate

becomes lower with the increase of Lewis number. Due to increase in thermophoresis parameter, both the temperature and nanoparticle volume fraction increase.

Singh et al. [38] summarizes the important results regarding the improvement in the thermo physical properties of nano uids. The in uence of important parameters like particle's ,base fluid type, temperature, additives and pH value has been considered.

Ali et al. [39] summarizes the current progress on the study of nanofluids, such as the fabrication procedures, stability evaluation mechanism, stability enhancement procedures, nanofluids thermophysical properties, and current commercialization challenges. Finally, the article identifies some possible opportunities for future research that can bridge the gap between in-lab research and commercialization of nanofluids.

Wong and Leon [40] focuses on presenting the broad range of current and future applications that involves nanofluids, emphasizing their improves heat transfer properties that are controllable and the specific characteristics that these nanofluids possess that make them suitable for such applications.

Matta and Gajjela [41] investigated the study of chemical reaction and convective boundary conditions on thermosolutal behavior of magneto hydrodynamic (MHD) uid ow above a stretching sheet. He discussed the physical importance of different flow variables such as the magnetic parameter, vortex viscosity parameter, order of chemical reaction, Prandtl, skin friction coefficient and Eckert number.

Gbadeyan et al. [42] studied the Boundary layer flow induced in a nanofluid due to a linearly stretching sheet in the presence of thermal radiation and induced magnetic field, following a similarity analysis of the transport equations. The transport model employed includes the effect of Brownian motion and thermophoresis. The analysis shows that temperature and concentration profiles in the respective boundary layers depend, besides the Prandtl and Lewis numbers, on five additional dimensionless parameters, namely a Brownian motion parameter Nb, a thermophoresis parameter Nt, a thermal radiation parameter Ra, a local magnetic field parameter Mx and a convection Biot number Bi. Bhattacharyya and Layek [43] presented a mathematical model of the steady boundary

layer flow of nanofluid due to an exponentially permeable stretching sheet with external magnetic field. He considered the effects of Brownian motion and thermophoresis on heat transfer and nanoparticle volume friction in this model.

Prasannakumara et al. [44] studied the effect of nonlinear thermal radiation on slip flow and heat transfer of fluid particle suspension with nanoparticles over a nonlinear stretching sheet immersed in a porous medium. Water is considered as a base fluid with dust particles along with suspended Aluminum Oxide(Al_2O_3) nanoparticles.

Bhargava and Chandra [45] studied the MHD boundary layer ow with heat and mass transfer of a nano uid with viscous dissipation over a non-linear stretching sheet embedded in a porous medium. The e ect of viscous dissipation, magnetic in uence parameter, permeability parameter, Brownian motion, Eckert number, Thermophoresis parameter, Prandtl number parameter, Dufour Lewis number and nano uid Lewis number are studied for velocity, temperature, concentration of salt and concentration of nanoparticles.

Daniel et al. [46] studied the combined effects of thermal stratification, applied electric and magnetic fields, thermal radiation, viscous dissipation and Joules heating on a boundary layer flow of electrical conducting nanofluid over a nonlinearly stretching sheet with variable thickness. The electrical conducting nanofluid particle fraction on the boundary is passively rather than actively controlled. The effects of the emerging parameters on the electrical conducting nanofluid velocity, temperature, and nanoparticles concentration volume fraction with skin friction, heat transfer characteristics are examined. It is observed that the variable thickness enhances the fluid velocity, temperature, and nanoparticle concentration volume fraction.

3 Research Gap Identification

In this research work, the concept of two dimensional boundary layer flows of nanofluids and magnetic nanofluids over linear and nonlinear stretching surfaces will be explored. Boundary layer flows of nanofluids are in the spotlight for the years and plenty of work has been done in this direction. Although, a number of concepts that remain untouched and there is a big gap that need to be filled. Some external factors such as shape and size of nanoparticles as well as external factors like pressure, temperature, reactions at the surfaces will give new ways to inspect in this areas. The numerical solutions of boundary layer flows of magnetic nanofluids and nanofluids will help in the understanding of the flow heat and mass transfer phenomena in a better way. The prime concern of this work is to introduce such concepts and theories that will helpful to give a perfect control on the flow characteristics of fluids and enhance mass and heat transfer rates.

4 Objectives

In this research work, we will focus ourselves in the understanding of boundary layer behavior of nanofluids flows over distinct geometrical surfaces. The prime objectives of present study will be :

- To develop mathematical models of the concerned physical problems by utilizing Buongiorno/ Tiwari and Das nanofluids model.

- To utilize the similarity transformations in order to find the self-similar solutions of the concerned problems.

- To exploit a numerical method in order to attained numerical solutions. To generate a computer code for the concerned numerical method in MATLAB/ MATHEMATICA/ MAPLE.

- To understand the influence of physical parameters like magnetic field strength, porous medium permeability, chemical reaction, radiations, thickness parameters on the nanofluid flows past exponential stretching sheet, curved stretching surfaces, nonlinear stretching surfaces and stretching surface with variable thickness.

- To draw these obtained results in graphical and tabular forms and discussed in detail.

- To comprehend the role of geometries (linear/ non-linear/ curved/ exponential) subjected to stagnation point flow of nanofluids.

5 Proposed Methodology

The physical problem that we have considered will transformed into partial differential equations with initial and boundary conditions in the beginning.Secondly, the obtained system of equations are dimensional hence similarity transformations will be introduced to make them dimensional free. Thirdly, the obtained non-dimensional highly nonlinear system of equations will be solved with the help of numerical techniques. After that, computer codes will be formulated for the used numerical methods in MATLAB /MATHEMATICA /MAPLE. The convergence and stability conditions will also be obtained for the applied numerical methods. Lastly, the achieved solutions will be presented in the forms of graph and tables.

6 Thesis Outline

Tentative chapter plan of this thesis is as under:

Chapter 1	Introduction
Chapter 2	Literature review
Chapter 3	Flow of magnetic nanofluids due to a linear stretching surface
Chapter 4	Flow of magnetic nanofluids due to a non-linear stretching surface
Chapter 5	Flow of magnetic nanofluids due to stretching surface of variable thickness
Chapter 6	Conclusions,implications and future work

Research Plan Schedule

Research activity	Progress in months					
	6	12	18	24	30	36
Course work and extensive literature survey	→	→				
Literature review and working on synopsis			→			
Literature review and working on synopsis				→		
Literature review and working on synopsis					→	
Work on first, second, third problem, publications/conference, Thesis writing and submission						→

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